Fresnel rhomb and other devices for handling and teaching polarization: inexpensive design

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Abstract: We demonstrate a pair of 90-45-45 prisms from binoculars appropriately attached to serve as Fresnel rhomb, i.e. achromatic quarter-wave plate. One and two Dove prisms with metallic reflection instead of TIR can work as half-wave plate and polarization rotator, respectively; both achromatic.

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1. Introduction

Fresnel rhomb is an important device to study Total Internal Reflection (TIR), circular polarization and other optical phenomena, [1,2]. It serves as a quarter-wave plate, which allows transformation of linear polarization into circular or elliptical and back. It uses the fact that in the vicinity of 50 degrees of incidence, one TIR between glass ($n \approx 1.5$) and air introduces s/p-phase difference about $\pi/4$, so that two sequential TIRs function as achromatic quarter-wave plate. R. Wood suggests, [3], making Fresnel rhomb by cutting at 54º two sides of a thick glass plate, then polishing it to a mediocre quality and attaching thin glass plates with Canada balsam to eliminate roughness of polishing. However, the price of Fresnel rhomb is typically 5 to 10 times larger than the price of a right-angle prism.

2. The devices suggested

We demonstrate here how a pair of 90-45-45 prisms (right-angle prisms) may be attached to function as Fresnel rhomb, see Figure 1. Immersion liquid was added between the prisms to eliminate parasitic reflections. Small adjustment of the incidence angle, about 5 to 10 degrees, allowed to reach quite good performance of the device as transformer of linear polarization into circular one, with the energy fraction of the "wrong" circular polarization less than 0.0015 at $\lambda = 0.6328$ μm.

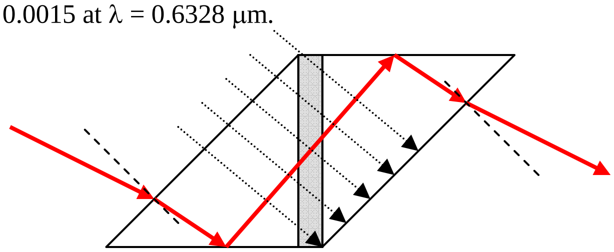

Figure 1. Modified Fresnel rhomb.

4 good right-angle prisms may be extracted from an old and otherwise dysfunctional binocular. It is worth noting that we actually used prisms from a binocular; they had antireflection coatings at the hypotenuse of the prisms and no coatings at short sides. Attempt to introduce light through short sides and use TIRs at hypotenuses yielded 1.41 times wider window, but did not give desired quarter-wave performance due to modification of phases by the coatings. The same device can serve as a demo of a thick flat glass plate (for the rays depicted by dotted arrows).

Dove prisms are conventionally used to rotate image. Most Dove prisms use TIR process for high-quality reflection of light. We suggest here to use <u>metallized reflecting surface</u>, Figure 2. Upper part of the prism is usually cut off. Important property of light reflection from perfect conductor is that the polarization vector is reflected, up to a sign, as an image of a physical object: if $\mathbf{n}$ is the vector of normal to the surface, then

$$\mathbf{E}_{ref} = -\mathbf{E}_{inc} + 2\mathbf{n}\,(\mathbf{n}\cdot\mathbf{E}_{inc}).$$

One can see that this prism serves as half-wave plate with respect to polarization, and one of the axes coincides with the normal to the mirror plane. Any prism may be used for that purpose, including (but not limited to) right-angle or equilateral.

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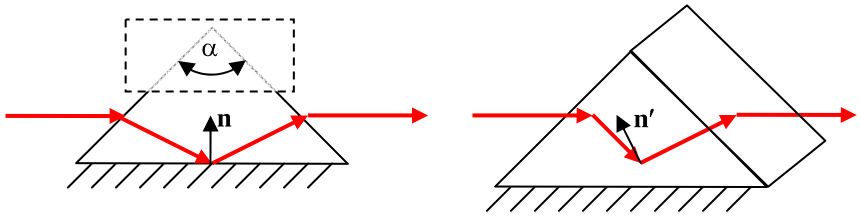

Figure 2. Dove prism modified by metallizing its reflecting surface, and tandem of two such prisms; **n** and **n′** are vectors normal to the mirrors.

Small value of the angle α at the top vortex leads, for a given base length, to relatively small window, but makes less of a change of intensity and polarization at the input and output sides due to Fresnel reflections. A pair (tandem) of two such prisms with an angle ψ between the normal vectors **n** and **n′** to the mirrors yields true rotation of both, the image carried by the beam and the polarization, by the angle 2ψ, so that change of ψ allows to continuously rotate the output polarization. It should be emphasized that the action of the above devices in Figure 2 is achromatic, even if the refractive index of the prism's material has arbitrary wavelength dependence. However, high conductivity of the metal and/or highly negative value of its permeability is crucial. While the process of metal deposition is reasonably inexpensive in the conditions of research laboratory or of industry, even simpler solution exists. One may use existing flat mirror, either with front or with back metallized surface, and attach it to the prism with the immersion liquid in-between.

3. Conclusion

To conclude, we suggested and demonstrated the performance of 3 inexpensive achromatic devices that function as 1) Fresnel rhomb, 2) thick parallel plate, 3) half-wave plate, and 4) polarization rotator.

7. References

1. M. Born, E. Wolf, Principles of Optics, 7-th ed., sect. 1.5.4, Cambridge U. Press, 1999.
2. E Hecht, Optics, 3-rd ed., sect. 8.7, Addison-Wesley, Reading, MA, 1998.
3. R. W. Wood, Physical Optics, 3-rd ed., Chap. IX, p. 354, OSA, Washington DC, 1988.

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Overview

Introduction: Teaching polarization to K-12 kids

Particular devices:

- Fresnel rhomb
- Dove prism, TIR with "smooth" transition
- Dove prism, TIR with "sharp" transition
- Aluminized Dove prism

What is to be learned:

a. Role of impedance vs. refractive index
b. Pseudo-parallel transport of polarization
c. Interference as enhancer of contrast

Conclusion

Teaching polarization to K – 12 audience:

mechanical analogs are very helpful.

Examples:

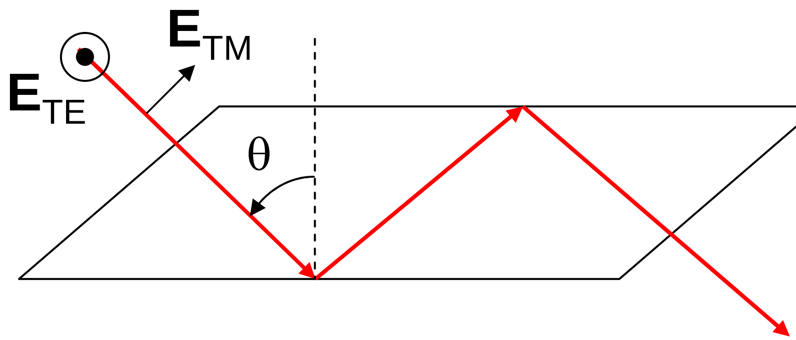
Motion of pendulum in (x, y)-directions
(see also next talk).

Waves on the rope easily demonstrate
linear, elliptical and circular polarization.

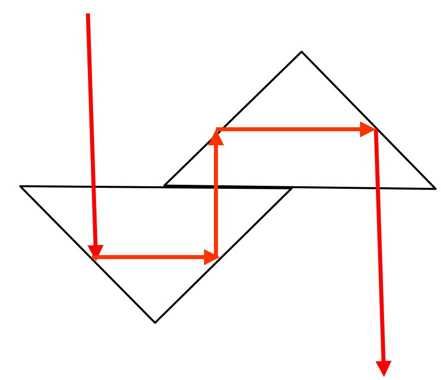
Change of polarization that accompanies the
change of propagation direction is instructive.

Fresnel rhomb is a prism

which performs 2 Total Internal Reflections (TIRs).
At an appropriate incidence angle (θ = 54° for n = 1.5), phase difference of reflection coefficients for TE and TM polarizations is $\pi/4$, so that two TIRs yield a quarter-wave plate action.



Commercial binoculars usually contain 2 pairs of 90°-45°-45° prisms, like these:

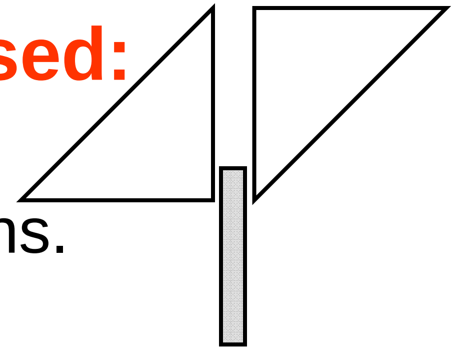


Statement made in this work:

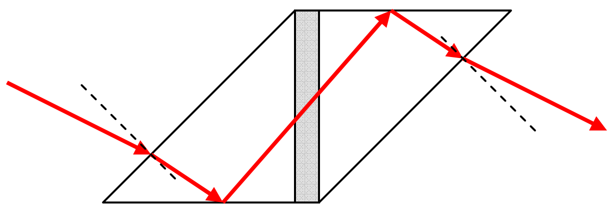
they may be composed
into a device functioning as a Fresnel rhomb.

Here is how they may be composed:

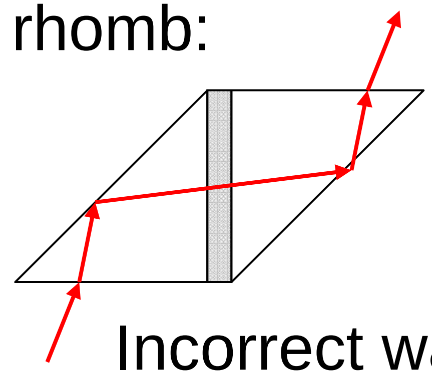
Immersion liquid (glycerin) helps
in eliminating unwanted Fresnel reflections.



Two ways to illuminate the resulting rhomb:

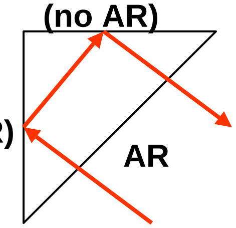


Correct way



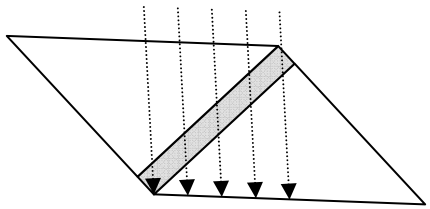
Incorrect way

Reason: prisms from binoculars have
Anti-Reflection (AR) coating at the hypotenuse,
but do not have AR at short sides.
In our device those coatings can spoil
the proper phase difference at TIRs.



<u>Very</u> thick parallel plate

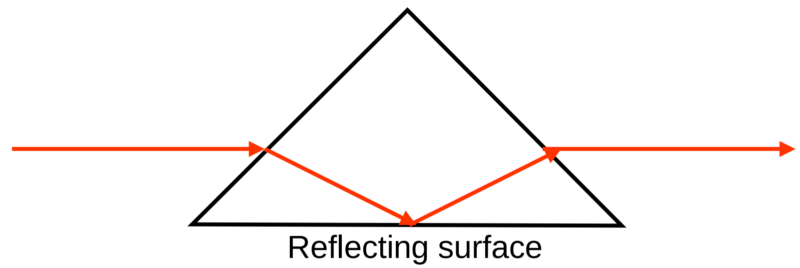
may be demonstrated via this device.



Huge lateral shift of the image is easily observed, when the “plate” is tilted.

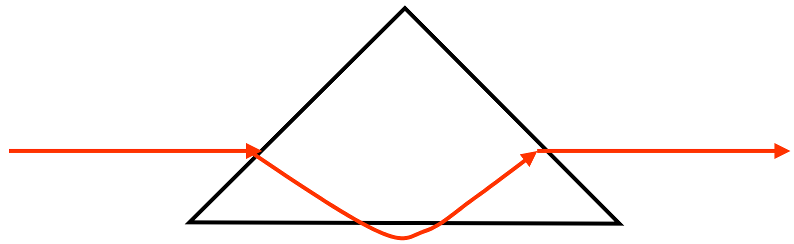
Observation should be made also of the role of the **immersion layer:** its absence results in metallic-looking shining TIR.

Dove prism performs one mirror reflection,
but combines it with such a (symmetric) refraction
that the central direction of the beam remains unchanged.



Q: What Dove prism does with the polarization ?
A: It depends on the type of reflecting surface.

Dove prism, TIR with "**smooth**" transition region.

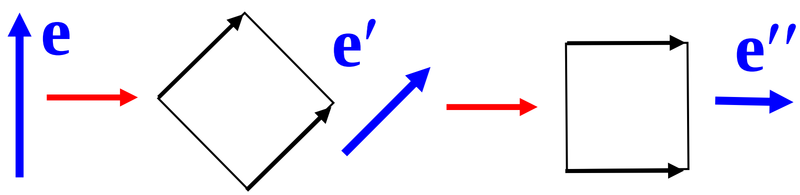


A hint in the direction of a "smooth" transition region is the Anti-Reflection (AR) coating of a prism from a binoculars.

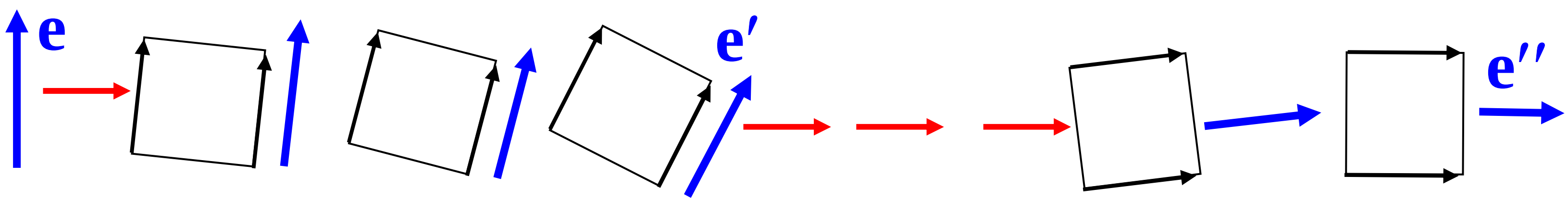
Evolution of polarization at propagation in "smooth" locally isotropic medium (Bortolotti, Rytov, Vladimirsky, Berry, Chiao, Tomita) is described below.

Step aside towards Berry's phase: quiz from undergraduate physics

Can one rotate polarization by absorbing polarizers ?



Two ideal polarizers: transmission $T = |t|^2 = 0.25.$



Ten (10) ideal polarizers:
transmission $T = |t|^2 = [\cos^2(90°/10)]^{10} = 0.78$, not bad !

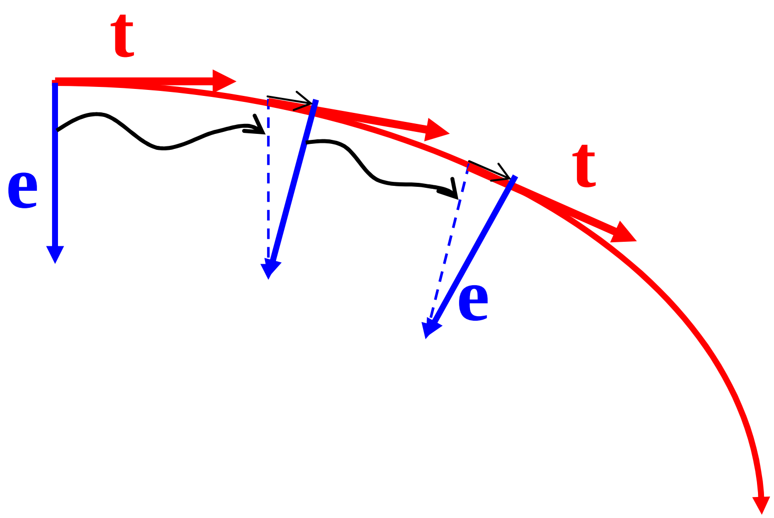
Infinite number of ideal polarizers: $T = |t|^2 \Rightarrow 1.$

Evolution of polarization in scalar medium along a curvilinear ray (Rytov-Berry-Chiao rotation).

1. Local isotropy, $\varepsilon_{ik} = \varepsilon_0 \delta_{ik}$, $\mu_{ik} = \mu_0 \delta_{ik}$, means that the polarization "does not want" to change.

2. Constant 3-D polarization vector **e** may become partially longitudinal – this is "forbidden" !

3. Solution: pseudo-parallel transport of polarization. **Subtract growing longitudinal part, but do not do anything else!** Equation for polarization vector **e**:

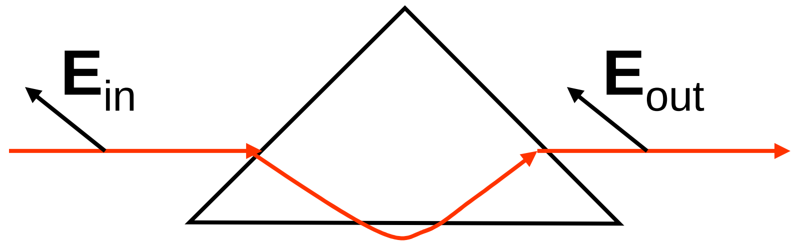


$$\frac{d\mathbf{e}}{dl} = -\mathbf{t}\left(\mathbf{e} \cdot \frac{d\mathbf{t}}{dl}\right),$$

l, length along the ray.

I prefer to call this "Rytov- Berry-Chiao **non**-rotation".

Dove prism, TIR with "**smooth**" transition region.

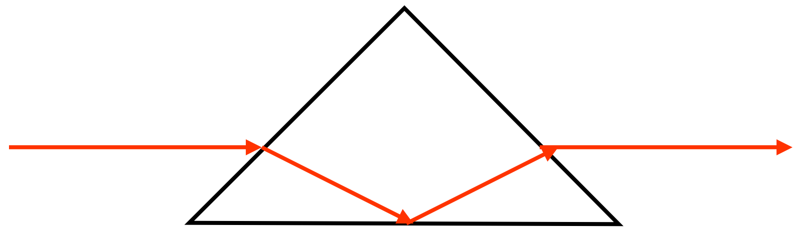


In the experiment with a particular prism from binoculars, **output polarization <u>reproduced</u> the input polarization** with quite good accuracy.

It was 100% for vertical and for horizontal polarizations. But **even for 45° input polarization**, the orientation of predominant component was **<u>preserved</u>**, and the intensity of "**wrong**" admixture was **<u>only 2.4%</u> - not bad!**

Note that the solid angle subtended by the **t**-vector at the unit sphere (Berry's phase) is zero here (planar trajectory of the ray).

Dove prism, TIR with "**sharp**" transition region.

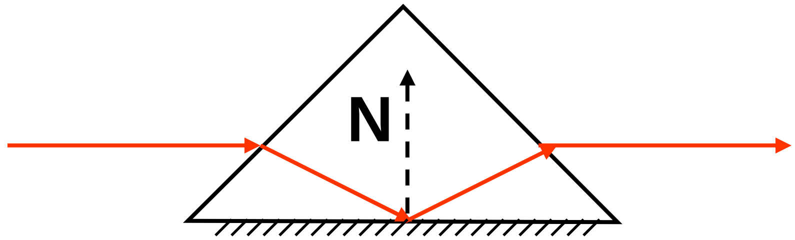


Reflecting surface:
TIR at a "sharp" boundary

In the experiment with a prism w/o AR at hypotenuse, **output polarization also reproduced the orientation of input polarization**, but the preservation of linear polarization was not precise.

It was 100% for vertical and for horizontal polarizations. For 45° input polarization, the orientation of predominant component was preserved, and the intensity of "**wrong**" admixture **was 10%** - also not bad, but considerable.

Dove prism, metallic reflection at the hypotenuse.



Aluminum-coated surface

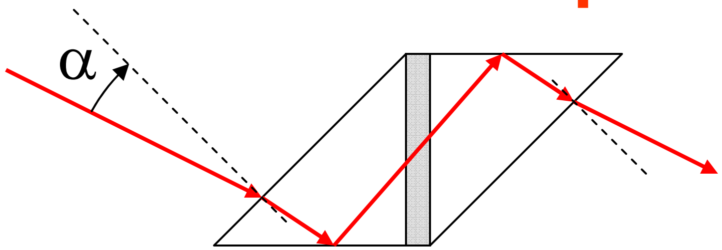
In the experiment with a prism aluminum-coated surface, orientation of output polarization was changed (approximately) as it should be at ideal metallic reflection:

$$\mathbf{E}_{out} = -\,\mathbf{E}_{in} + \mathbf{N}\,(\mathbf{N}\cdot\,\mathbf{E}_{in}),$$

or as if transformed by an achromatic half-wave plate.

For 45° input polarization, the orientation of predominant component <u>was changed to orthogonal</u>, but the intensity of “wrong” admixture was rather large: 27% at λ=632.8 nm.

Experiments with the "composite" Fresnel rhomb



At α=0 (normal incidence) and 45° orientation of the input polarization, the output polarization was not perfectly circular:

$$|(\mathbf{E}_{out}\cdot\mathbf{e}_{\boldsymbol{x}})|^2 = 0.67, \quad |(\mathbf{E}_{out}\cdot\mathbf{e}_{\boldsymbol{y}})|2 = 0.33.$$

At $\alpha \approx 27°$ (θ corresponds to optimum, 54°)

$$|(\mathbf{E}_{out}\cdot\mathbf{e}_{\boldsymbol{x}})|^2 = 0.58, \quad |(\mathbf{E}_{out}\cdot\mathbf{e}_{\boldsymbol{y}})|^2 = 0.42.$$

How good is the degree of circular polarization here ?

Remarkable properties of interference

Let us take two waves, one with **intensity** $|E_1|^2 = \underline{\mathbf{1}}$,
and the other with **intensity** $|E_2|^2 = \rho$, e.g. $\rho = \underline{\mathbf{0.01}}$.
It means that the amplitudes are

$$E_1 = \underline{1} \text{ and } E_2 = \sqrt{\rho} = \underline{0.1},$$

When the two waves interfere constructively,

$$E_{max} = E_1 + E_2 = 1 + \sqrt{\rho} = \underline{1.1}.$$

When the waves interfere destructively,

$$E_{min} = E_1 + E_2 = 1 - \sqrt{\rho} = \underline{0.9}$$

Corresponding intensities are

$$|E_{max}|^2 = \underline{1.21}, \qquad |E_{min}|^2 = \underline{0.81}.$$

Full scale of the intensity difference is 40% !!!

How good is our circular polarization ?

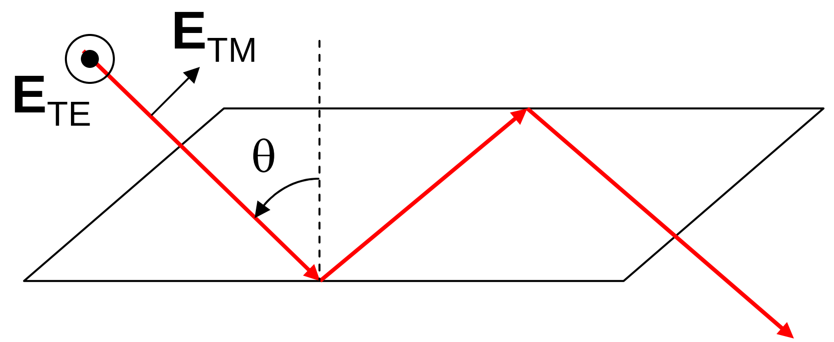
Ratio of intensities R =0.33 / 0.67 means
that the intensity of the admixture of the “wrong”
circular component constituted
$\rho = (1-\sqrt{R})^{1/2} / (1+\sqrt{R})^{1/2} = 0.03$ (quite small **!**)

Ratio of intensities R = 0.42 / 0.58 means
that the intensity of the admixture of the “wrong”
circular component constituted
$\rho = (1-\sqrt{R})^{1/2} / (1+\sqrt{R})^{1/2} = 0.0064$ - really small **!!!**

Impedance Z versus refractive index n.

$$Z = (\mu / \varepsilon)^{1/2}, \quad Z_{vac} = 377 \text{ Ohm};$$
$$n = (\mu\varepsilon / \mu_{vac}\varepsilon_{vac})^{1/2}, \quad n_{vac} = 1.$$

Let us make an **unusual Fresnel rhomb:**
in radiofrequency range, in a way that
$\mu = 1.5\mu_{vac}$, $\varepsilon = 1.5\varepsilon_{vac}$, $Z = Z_{vac}$, but n =1.5 (like glass).
Then the trajectory of TIR-reflected rays will be the same as for glass.
However, here will be no reason for TE polarization to have any phase delay or advance in comparison with TM polarization. Hence this rhomb will not work as quarter-wave plate !



Statement:

The work of Fresnel rhomb is based on the
step of impedance Z, and not of the refractive index n!!!

Conclusion

1. **Fresnel rhomb may be designed out of two prisms (e.g. from binoculars).**

2. **Variants of Dove prism yield radically different evolution of polarization.**

3. **Playing with polarization is great fun !**